Topic: Extrema on a closed interval

Question: Find absolute extrema of the function on the closed interval.

$$f(x) = x^3 - 2x$$

on the interval $[-2,1]$

Answer choices:

A Minimum at $(1,1)$ Maximum at $(-2,10)$

B Minimum at $(-2,-4)$ Maximum at $\left(-\sqrt{\frac{2}{3}}, \frac{4}{3}\sqrt{\frac{2}{3}}\right)$

C Minimum at $\left(\pm\sqrt{\frac{2}{3}}, 0\right)$ Maximum at $(-2,10)$

D Minimum at $\left(-\sqrt{\frac{2}{3}}, -\frac{8}{3}\sqrt{\frac{2}{3}}\right)$ Maximum at $\left(\sqrt{\frac{2}{3}}, \frac{8}{3}\sqrt{\frac{2}{3}}\right)$

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Solution: B

In order to find the absolute extrema, we first need to find the critical numbers, where the extrema *could* occur. Critical numbers occur where the derivative of the function is equal to 0.

$$f'(x) = 3x^2 - 2$$

$$3x^2 - 2 = 0$$

$$x^2 = \frac{2}{3}$$

$$x = \pm\sqrt{\frac{2}{3}}$$

Absolute extrema could also occur at the endpoints, so now we check the value of the function at the endpoints and the critical numbers.

Extrema at	$x = -2$	$x = -\sqrt{\frac{2}{3}}$	$x = \sqrt{\frac{2}{3}}$	$x = 1$
$f(x)$	-4	$\frac{4}{3}\sqrt{\frac{2}{3}}$	$-\frac{4}{3}\sqrt{\frac{2}{3}}$	-1

The extrema of the function are the following:

Absolute minimum at $(-2, -4)$

Absolute maximum at $\left(-\sqrt{\frac{2}{3}}, \frac{4}{3}\sqrt{\frac{2}{3}}\right)$

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Topic: Extrema on a closed interval

Question: What are the extrema of the function over the interval $[0,3]$?

$$f(x) = x^2 - 4x$$

Answer choices:

A Global minimum at $(3, -3)$; Global maximum at $(2, -4)$.

B Global maximum at $(2, -4)$; Global maximum at $(3, -3)$.

C Global minimum at $(0,0)$; Global maximum at $(2, -4)$.

D Global minimum at $(2, -4)$; Global maximum at $(0,0)$.

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Solution: D

We'll start by finding critical numbers, which means we'll take the first derivative of the function.

$$f(x) = x^2 - 4x$$

$$f'(x) = 2x - 4$$

$$f'(x) = 2(x - 2)$$

Now set the derivative equal to 0 and solve for x.

$$0 = 2(x - 2)$$

$$x - 2 = 0$$

$$x = 2$$

There are three critical numbers at $x = 0$, $x = 2$ and $x = 3$. Remember that the endpoints need to be considered.

Now take a point on either side of the critical number $x = 2$ (we will use $x = 1$ and $x = 5/2$) and substitute these into the first derivative.

$$f'(1) = 2(1) - 4$$

$$f'(1) = 2 - 4$$

$$f'(1) = -2$$

and

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$$f'\left(\frac{5}{2}\right) = 2\left(\frac{5}{2}\right) - 4$$

$$f'\left(\frac{5}{2}\right) = 5 - 4$$

$$f'\left(\frac{5}{2}\right) = 1$$

We can see that on the left-hand side of $x = 2$ the function is negative, so it's decreasing. On the right-hand side of $x = 2$ the function is positive, so it's increasing. This means that at $x = 2$ there's a global minimum. Remember when the function goes from decreasing to increasing, a minimum exists.

Next find the y-value of the endpoints by substituting back into the original function.

At $x = 0$,

$$f(x) = x^2 - 4x$$

$$f(0) = (0)^2 - 4(0)$$

$$f(0) = 0$$

This gives us the point $(0,0)$.

At $x = 3$,

$$f(x) = x^2 - 4x$$

$$f(3) = (3)^2 - 4(3)$$

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$$f(3) = 9 - 12$$

$$f(3) = -3$$

This gives us the point $(3, -3)$.

Because the y-value of the point $(0,0)$ is higher than the y-value of the point $(3, -3)$, this means that $(0,0)$ is a global maximum.

Now find the y-component of $x = 2$.

$$f(2) = (2)^2 - 4(2)$$

$$f(2) = 4 - 8$$

$$f(2) = -4$$

This gives us the point $(2, -4)$.

The function $f(x) = x^2 - 4x$ on the interval $[0,3]$ has a global minimum at the point $(2, -4)$ and a global maximum at the point $(0,0)$.

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Topic: Extrema on a closed interval

Question: What are the extrema of the function over the interval $[0,2]$?

$$f(x) = x^3 - 3x$$

Answer choices:

A Global minimum at $(1, -2)$; Global maximum at $(2,2)$.

B Global minimum at $(2,2)$; Global maximum at $(1, -2)$.

C Global minimum at $(-1,2)$; Global maximum at $(2,2)$.

D Global minimum at $(2,2)$; Global maximum at $(-1,2)$.

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Solution: A

We'll start by finding critical numbers, which means we'll take the first derivative of the function.

$$f(x) = x^3 - 3x$$

$$f'(x) = 3x^2 - 3$$

$$f'(x) = 3\left(x^2 - 1\right)$$

$$f'(x) = 3(x + 1)(x - 1)$$

Now set the derivative equal to 0 and solve for x.

$$0 = 3(x + 1)(x - 1)$$

$$x = -1, 1$$

The point $x = -1$ is outside the interval $[0,2]$, so we only need to consider $x = 1$. Therefore, there are three critical numbers at $x = 0, 1, 2$. Remember that the endpoints need to be considered.

Now take a point on either side of the critical number $x = 1$ (we will use $x = 1/2$ and $x = 3/2$) and substitute these into the first derivative.

$$f'\left(\frac{1}{2}\right) = 3\left(\frac{1}{2}\right)^2 - 3$$

$$f'\left(\frac{1}{2}\right) = 3\left(\frac{1}{4}\right) - 3$$

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$$f'\left(\frac{1}{2}\right)=\frac{3}{4}-3$$

$$f'\left(\frac{1}{2}\right)=\frac{3-12}{4}$$

$$f'\left(\frac{1}{2}\right)=-\frac{9}{4}$$

and

$$f'\left(\frac{3}{2}\right)=3\left(\frac{3}{2}\right)^2-3$$

$$f'\left(\frac{3}{2}\right)=3\left(\frac{9}{4}\right)-3$$

$$f'\left(\frac{3}{2}\right)=\frac{27}{4}-3$$

$$f'\left(\frac{3}{2}\right)=\frac{27-12}{4}$$

$$f'\left(\frac{3}{2}\right)=\frac{15}{4}$$

We can see that on the left-hand side of $x=1$ the function is negative, so it's decreasing. On the right-hand side of $x=1$ the function is positive, so it's increasing. This means that at $x=1$ there's a global minimum. Remember when the function goes from decreasing to increasing, a minimum exists.

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Next find the y-value of the endpoints by substituting back into the original function.

At $x = 0$,

$$f(x) = x^3 - 3x$$

$$f(0) = (0)^3 - 3(0)$$

$$f(0) = 0$$

This gives us the point $(0,0)$.

At $x = 3$,

$$f(x) = x^3 - 3x$$

$$f(2) = (2)^3 - 3(2)$$

$$f(2) = 8 - 6$$

$$f(2) = 2$$

This gives us the point $(2,2)$.

Because the y-value of the point $(2,2)$ is higher than the y-value of the point $(0,0)$, this means that $(2,2)$ is a global maximum.

Now find the y-component of $x = 1$.

$$f(1) = (1)^3 - 3(1)$$

$$f(1) = 1 - 3$$

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$$f(1) = -2$$

This gives us the point $(1, -2)$.

The function $f(x) = x^3 - 3x$ on the interval $[0,2]$ has a global minimum at the point $(1, -2)$ and a global maximum at the point $(2,2)$.

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